Modulation Instability and Tunable Raman Gain in Mid-IR Waveguides

Pablo Fierens
Our group

• Instituto Tecnológico de Buenos Aires
  • Private university
  • ~ 3500 students
  • Focus on engineering
  • 3 PhD programs
  • Several graduate programs
  • 10 undergrad degrees
Our group

Optics

Nonlinear optics

Noise in nonlinear systems

Robust Computing

Stochastic Resonance

Memristors

Wireless Sensors

Security

Comm.

Ortega et al., J. Inf. Secur., 2014
Grisales et al., IEEE ARGENCON, 2018
Fierens, Int. J. Wildland Fire, 2009

Pessac et al., CNSNS., 2015
Patterson et al., Physica A, 2010

Cisternas et al., Materials, subm.
Patterson et al., APL., 2013
Light sources in the mid-IR

- Molecular fingerprint region
- Need for broadband and intense light sources
  - A common approach: supercontinuum generation (SC)
    - CO\textsubscript{2} laser as pump (10 µm, 5 µm from SHG) on a chalcogenide waveguide?
- Correlated photon pairs in the atmospheric windows (3-4 and 8-12 µm) for quantum information transmission
  - Parametric processes of sum/difference of frequencies in crystals (GaSe, AgGaSe\textsubscript{2}) with second order nonlinear susceptibility \(\chi_2\)?

This talk
Nonlinear optics

- What are we interested in?
  - Supercontinuum generation in the mid IR  (Dudley, Genty & Coen, Rev. Mod. Phys, 2006)
    - Simulation: 200 fs pump pulse + noise
    - Chalcogenide glass fiber
    - $P_0 = 1 \text{kW}, \lambda_0 = 5 \mu\text{m}$
Nonlinear optics

- What are we interested in?
  - Intense pulses - rogue waves
    - Simulation: 200 fs pump pulse + noise
    - Chalcogenide glass fiber
    - $P_0 = 1$ kW, $\lambda_0 = 5$ μm

(Solli et al., Nature, 2007)

Grosz et al., EECOS 2015
Nonlinear optics

- What are we interested in?
  - Intense pulses - rogue waves

Grosz et al., EECOS 2015
Nonlinear optics

- What are we interested in?
  - Supercontinuum generation in the mid IR
  - Intense pulses - rogue waves
  - Parametric amplification  (Stolen & Bjorkholm, IEEE J. Quantum Electron., 1982)
Nonlinear optics

- What are we interested in?
  - Supercontinuum generation in the mid IR
  - Intense pulses - rogue waves
  - Parametric amplification

Modulation instability

Propagation of a CW in an optical fiber is unstable

breaks up into pulses

Benjamin & Feir, J. Fluid Mech., 1967
Shabat & Zakharov, JETP, 1972
Tai, Hasegawa & Tomita, PRL, 1986
Potasek, Optics Lett., 1987
Modulation instability

- 40 years of research! Should I end my talk now?
  - Most of the analyses of MI do not include all details relevant to optical fibers.
  - Not a lot of work on (quasi-)analytical approaches to the interaction of noise and nonlinearity in MI

- Coming next...
  - A complete analysis of the spectral evolution of a perturbation to a CW
  - Analytical results on input noise + MI
Propagation in optical fibers

- **Generalized Nonlinear Schrödinger Equation (GNLSE)**

\[
\frac{\partial A}{\partial z} - i\hat{\beta} A = i\hat{\gamma} A(z, T) \int_{-\infty}^{+\infty} R(T')|A(z, T - T')|^2 dT'
\]

- **Dispersion**

\[
\hat{\beta} = \sum_{m \geq 2} \frac{i^m}{m!} \beta_m \frac{\partial^m}{\partial T^m}
\]

- **Nonlinearity**

\[
\hat{\gamma} = \sum_{n \geq 0} \frac{i^n}{n!} \gamma_n \frac{\partial^n}{\partial T^n}
\]

- **Raman scattering**

\[
R(T) = (1 - f_R)\delta(T) + f_R h(T)
\]
Perturbation to the stationary solution

\[ A(z, T) = (\sqrt{P_0} + a)e^{i\gamma_0 P_0 z} = A_s + ae^{i\gamma_0 P_0 z} \]

- Input power: \( P_0 \)
- Perturbation: \( a(z; T) \)

- Linear terms in the frequency domain

\[
\frac{\partial \tilde{a}(z, \Omega)}{\partial z} + \tilde{N}(\Omega)\tilde{a}(z, \Omega) = \tilde{M}(\Omega)\tilde{a}^*(z, -\Omega)
\]

- Frequency: \( \Omega = \omega - \omega_0 \)
- \( \tilde{N}(\Omega) = -i \left[ \tilde{\beta}(\Omega) + P_0 \tilde{\gamma}(\Omega) \left( 1 + \tilde{R}(\Omega) \right) - P_0 \gamma_0 \right] \)
- \( \tilde{M}(\Omega) = iP_0 \tilde{\gamma}(\Omega)\tilde{R}(\Omega) \)
Perturbation to the stationary solution

\[ \frac{\partial \tilde{a}(z, \Omega)}{\partial z} + \tilde{N}(\Omega)\tilde{a}(z, \Omega) = \tilde{M}(\Omega)\tilde{a}^*(z, -\Omega) \]

- Ansatz: \( a(z, \Omega) = D \exp(iK(\Omega)z) \)

\[ K(\Omega) = -\tilde{B}(\Omega) \pm K_D(\Omega) = -\tilde{B}(\Omega) \pm \sqrt{\tilde{B}^2(\Omega) - \tilde{C}(\Omega)} \]

- \( \tilde{B}(\Omega) \) and \( \tilde{C}(\Omega) \) are complex functions of the parameters

Fierens et al., ICAND 2016
Modulation instability gain

• Only self-steepening: \( \gamma_1 = \gamma_0 \tau_{sh}, \quad \gamma_n = 0 \) for \( n \geq 2 \)

\[
K(\Omega) = \bar{\beta}_o + P_0 \gamma_0 \tau_{sh} \Omega (1 + \bar{R}) \pm \sqrt{(\bar{\beta}_e + 2 \gamma_0 P_0 \bar{R}) \bar{\beta}_e + P_0^2 \gamma_0^2 \tau_{sh}^2 \Omega^2 \bar{R}^2}
\]

• Well-known facts about MI gain = \( 2 \text{Im}\{K(\Omega)\} \):
  
  ▪ It does not depend on odd terms of the dispersion relation
  ▪ Self-steepening enables a gain even in a zero-dispersion fiber
  ▪ In the large power limit, it is independent of the dispersion and it is dominated by Raman:

\[
|g(\Omega)| \approx 2P_0 \gamma_0 \tau_{sh} |\Omega| \cdot |\text{Im}\{\bar{R}(\Omega)\}|
\]
Spectral evolution

\[ \tilde{a}(z, \Omega) = e^{-i\tilde{B}(\Omega)z} \cdot \tilde{M}(\Omega) \sin(K_D(\Omega)z) \tilde{a}^*(0, -\Omega) + e^{-i\tilde{B}(\Omega)z} \cdot \frac{K_D(\Omega)}{K_D(\Omega)} \cdot [K_D(\Omega) \cos(K_D(\Omega)z) - (\tilde{N}(\Omega) - i\tilde{B}(\Omega)) \sin(K_D(\Omega)z)] \tilde{a}(0, \Omega) \]

- Interaction between \( \tilde{a}(z, \Omega) \) and \( \tilde{a}(z, -\Omega) \) due to the nonlinearity

- \( a(0, T) \in \mathbb{R} \Rightarrow \tilde{a}(z, \Omega) = \tilde{H}(\Omega, z)\Lambda(\Omega) \)
Noise-only

\[ \tilde{a}(0, \Omega) \sim \mathcal{CN}(0, \sigma^2) \rightarrow \tilde{a}(z, \Omega) \sim \mathcal{CN}(0, \sigma_{\tilde{a}}^2) \]

\[ \rightarrow |\tilde{a}(z, \Omega)| \sim \text{Rayleigh}(\sigma_{\tilde{a}}) \rightarrow |\tilde{a}(z, \Omega)|^2 / \sigma_{\tilde{a}}^2 \sim \chi^2_2 \]

- \( \sigma_{\tilde{a}}^2(z, \Omega) \) can be easily computed from previous equations
Noise-only
Noise-only

[Fierens et al., ICAND 2016]
• A more interesting case: additive white Gaussian noise

\[ a(0, \Omega) = \tilde{s}(\Omega) + \eta(\Omega), \quad \eta(\Omega) \sim \mathcal{CN}(0, \sigma_\eta^2) \]

• Relevant for controlling the generation of rogue waves
  - Solli, Ropers & Jalali, PRL, 2008
  - Sørensen et al., JOSA B, 2012

• We developed analytical expressions for some relevant metrics of the resulting spectrum
Noisy input

• Coherence (Dudley, Genty & Coen, Phys. Rev. Mod., 2006)

\[ g_{12}(z, \Omega) = \frac{< \hat{a}_k^*(z, \Omega) \hat{a}_l(z, \Omega) >_{k \neq l}}{\sqrt{< |\hat{a}_k(z, \Omega)|^2 > < |\hat{a}_l(z, \Omega)|^2 >}} \]

• Characterizes shot-to-shot fluctuations in the phase of supercontinuum spectra
Noisy input

- Coherence
  - ✓ Standard Single Mode Fiber (SSMF)
  - ✓ 1 W pump at 1550 nm
  - ✓ 1 mW power seeds at 31 and 46 GHz
Noisy input

- A simple case:
  - One-sided seed, $\bar{s}(\Omega) = 0$ for $\Omega < 0$
  - No self-steepening and no Raman
- Small $z$
  \[
g_{12}(z, \Omega) \approx 1 - \left( 1 + \left( \frac{z}{L_{NL}} \right)^2 \right) \frac{\sigma^2}{|\bar{s}(|\Omega|)|^2} \quad \Omega > 0
  \]
  \[
g_{12}(z, \Omega) \approx 1 - \left( 2 + \left( \frac{L_{NL}}{z} \right)^2 \right) \frac{\sigma^2}{|\bar{s}(|\Omega|)|^2} \quad \Omega < 0
  \]
- Large $z$
  \[
g_{12}(z, \Omega) \approx 1 - 2 \frac{\sigma^2}{|\bar{s}(|\Omega|)|^2}
  \]
Modulation instability

• Analytical expressions for the spectral evolution of a perturbation to a continuous pump propagating in an optical fiber, including all relevant details

• Analytical results for some metrics of supercontinuum generation, such as coherence, for noisy inputs

• Problems:
  ▪ Undepleted pump $\Rightarrow$ valid for short distances
  ▪ Disregards cascading four-wave mixing effect
Higher-order perturbation

• The analysis can be extended to higher-order perturbation

• If $|\tilde{a}(0, \Omega)|^2 \geq s$, the first order solution can be written as

$$< |\tilde{a}_1(z, \Omega)|^2 > \approx s + (e^{2g(\Omega)z} - 1)|A_1(\Omega)|^2 s$$

• This first order solution motivates the ansatz

$$\tilde{a}(z, \Omega) \approx \sqrt{s} e^{i\phi_0(z, \Omega)} + \sum_{n=1}^{\infty} (e^{G_n(\Omega)z} - 1)A_n(\Omega)\sqrt{s^n} e^{i\phi_n(z, \Omega)}$$
Higher-order perturbation

- After some lengthy manipulations, we arrive at the following expressions:

\[ G_n(\Omega) = \max_u G_1(u) + G_{n-1}(u - \Omega) \]

\[ < |A_n(\Omega)|^2 > = \frac{\alpha^{n-1}\gamma^2(\Omega) \left[ |\tilde{B}(-\Omega) - iG_n(\Omega)|^2 + \gamma^2(-\Omega) \right]}{\left| (\tilde{B}(\Omega) + iG_n(\Omega)) (\tilde{B}(\Omega) - iG_n(\Omega)) - \gamma(\Omega)\gamma(-\Omega) \right|^2} \]

- The first equation describes the cascading effect of four-wave mixing.
Higher-order perturbation

Bonetti et al., CNSNS, 2019
Higher-order perturbation

- 770 m-long, dispersion-stabilized Highly-Nonlinear Fiber (HNLF)
- CW 30-dBm pump laser at 1590.4 nm
Higher-order perturbation

✓ Dispersion-stabilized Highly-Nonlinear Fiber (HNLF)
✓ CW 30-dBm pump laser at 1590.4 nm
Modulation instability: Power cutoff

• In the absence of Raman scattering, there is a limit in the MI gain due to self-steepening (Shukla & Rasmussen, Opt. Lett., 1986, De Angelis et al., JOSA B, 1996)

• In general, the MI gain vanishes whenever the power is equal to

\[ P_\pm = \hat{P}(\Omega) \times \left( 1 \pm \sqrt{1 - \tau_{sh}^2 \Omega^2} \right) \]

\[ \hat{P}(\Omega) = -\frac{\beta_e(\Omega)}{\gamma_0 \tau_{sh}^2 \Omega^2} \]

Hernandez et al., IEEE Photonics J., 2017
Modulation instability: Power cutoff

- In the absence of Raman scattering, there is a limit in the MI gain due to self-steepening

- MI gain versus pump power
- $\beta_2 = -1 \text{ ps}^2/\text{km}$
- $\beta_4 = -16, -8, +8, +16 \times 10^{-4} \text{ ps}^4/\text{km}$
- $\gamma_0 = 100 \text{ 1/(W km)}$
- $\lambda_0 = 5 \mu\text{m}$

- Gain region
- Location of the maximum MI gain

Hernandez et al., IEEE Photonics J., 2017
Modulation instability: Power cutoff

- In the presence of Raman scattering, there is still gain after the power cutoff

✓ MI gain versus normalized pump power
✓ $\beta_2 = -50$ ps$^2$/km
✓ $\gamma_0 = 100$ $1/(W\ km)$
✓ $\lambda_0 = 5$ $\mu m$

Sánchez et al., JOSA B, 2018
Tunable Raman gain

• Gain beyond the cutoff power can be tuned using the pump power

• Possible applications
  ▪ Mid-IR fiber Raman lasers
  ▪ Mid-IR supercontinuum generation

• Why mid-IR?
  ▪ Cutoff power $\propto \tau_{sh}^{-2}$ and $\tau_{sh} \approx \omega_0^{-1}$

Sánchez et al., JOSA B, 2018
Tunable Raman gain

- Gain beyond the cutoff power can be tuned using the pump power
Tunable Raman gain

- Gain beyond the cutoff power can be tuned using the pump power

Central frequency (deviation from Raman peak) as a function of the normalized power

Sánchez et al., JOSA B, 2018
Tunable Raman gain

- Tuning a filter central frequency

✓ Same pump and two seeds (simulations)
✓ $p = 1.1$ (top) and $p = 3.0$ (bottom)
✓ Output spectra after 3.6 mm

Sánchez et al., JOSA B, 2018
Anti-Stokes Raman gain

- Raman gain appears only in the Stokes side of the pump (lower frequencies)
  - CW pump + noise (simulations)
  - Normal regime: $\beta_2 = +50 \text{ ps}^2/\text{km}$
  - $\gamma_0 = 100 \text{ 1/(W km)}$
  - $\lambda_0 = 5 \mu\text{m}$

- Initially, there is no gain in the Stokes side
- Gain in the Stokes side appears as a consequence of four-wave mixing

Sánchez et al., JOSA B, 2018 B
Anti-Stokes Raman gain

- Raman gain appears only in the Stokes side of the pump (lower frequencies)

- CW pump + seeds (simulations)
- Normal regime: $\beta_2 = +50 \text{ ps}^2/\text{km}$
- $\gamma_0 = 100 \text{ 1/(W km)}$
- $\lambda_0 = 5 \mu\text{m}$

- Photon number is a conserved quantity

Sánchez et al., JOSA B, 2018
Anti-Stokes Raman gain

• Raman gain appears only in the Stokes side of the pump (lower frequencies)

• However, in the case of tunable Raman gain, it appears also in the anti-Stokes band (higher frequencies)

• Both gain sidelobes are Raman-shaped

• It is a pseudo-parametric gain

Sánchez et al., JOSA B, 2018 B
Ant-Stokes Raman gain

- Raman gain appears only in the Stokes side of the pump (lower frequencies)

✓ CW pump + seeds (simulations)
✓ Anomalous regime: $\beta_2 = -50 \text{ ps}^2/\text{km}$
✓ $\gamma_0 = 100 \text{ 1/(W km)}$
✓ $\lambda_0 = 5 \text{ \textmu m}$
✓ $p = 1.1$

- Both seeds grow simultaneously

Sánchez et al., JOSA B, 2018 B
Characterization of Raman

- Raman delayed response

\[
\frac{\partial A}{\partial z} - i\beta A = i\gamma A(z, T) \int_{-\infty}^{+\infty} R(T')|A(z, T - T')|^2 dT'
\]

\[
R(T) = (1 - f_R)\delta(T) + f_R h(T)
\]

- \( h(t) \) can be estimated from the Raman gain (\( \propto f_R n_2 \text{Im}\{\tilde{h}(\Omega)\} \)) and using the Kramer-Kronig relations

- \( f_R \) can be estimated given independent measurements of the Raman gain and \( n_2 \)
Characterization of Raman

- Raman delayed response

\[ R(T) = (1 - f_R)\delta(T) + f_R h(T) \]

- \( f_R \) can also be estimated from independent measurements of the Raman gain and the differential scattering cross-section
  - This approach was used in Stolen et al., JOSAB, 1989, to fix the value \( f_R = 0.18 \) used for silica fibers

- Time-resolved Z-scan can be used to measure both \( f_R \) and \( h(t) \)
  - Error > 25% for \( f_R \) (Smolorz et al., J. Non-Crystalline Solids, 1999)
Characterization of Raman

- Raman gain beyond the cutoff power enables another path to estimate $f_R$
  - The position of the peak of the Raman gain depends $f_R (p = 10)$
  - Equations relating the position of the peak with $f_R$

Sánchez et al., Opt. Lett, 2019
Extension of the GNLSE

• The Generalized Nonlinear Schrödinger Equation (GNLSE) does not preserve the photon number when a general nonlinear coefficient $\tilde{\gamma}(\Omega)$ is used.

• Waveguides based on new metamaterials require the use strongly frequency-dependent $\tilde{\gamma}(\Omega)$: some waveguides present a zero-nonlinearity wavelength.

• We developed a new extension to the GNLSE that is physically meaningful and can model the propagation in new materials.

Extension of the GNLSE

- Following a quantum mechanical approach, the equation can be derived (dispersionless case) from the mean evolution of the Schrödinger equation (Lai & Haus, Phys. Rev. A, 1989)

\[
\frac{\partial}{\partial z} |\psi> = i\hat{H} |\psi>
\]

\[
\hat{H} = \iiint \frac{\kappa}{2} \hat{a}^\dagger_{\omega_1} \hat{a}^\dagger_{\omega_2} \hat{a}_{\omega_1-\mu} \hat{a}_{\omega_2-\mu} d\omega_1 d\omega_2 d\mu
\]

- \(\hat{A}_\omega \propto \hat{a}_\omega \sqrt{\omega_0 + \omega}\)

- \(\kappa\) is related to the third order susceptibility

- Restriction on frequency dependence: hermiticity requires \(\kappa_{\omega_1,\omega_2,\omega_3,\omega_4} = \kappa_{\omega_4,\omega_3,\omega_2,\omega_1}\)
Extension of the GNLSE

• Hard-to-conduct measurements of the four-frequency dependence

• Idea: use generalized Miller’s rule

\[ \chi^{(3)}_{\omega_1, \omega_2, \omega_3, \omega_4} \propto \chi^{(1)}_{\omega_1} \chi^{(1)}_{\omega_2} \chi^{(1)}_{\omega_3} \chi^{(1)}_{\omega_4} \]

• Motivated by this idea:

\[ r_\omega \propto 4\sqrt{\gamma(\omega) \times (\omega_0 + \omega)} \]

\[ \kappa_{\omega_1, \omega_2, \omega_3, \omega_4} = \text{Re}(r_{\omega_1} r_{\omega_2} r_{\omega_3} r_{\omega_4}) \]
Extension of the GNLSE

✓ 100 ns pulse
✓ Dispersionless
✓ $\gamma_0 = 1.2 \times 10^{-3}$ 1/(W km)
✓ $\lambda_0 = 1550$ nm
✓ $P_0 = 200$ W

Bonetti et al., JOSA B, submitted
Summary

- Analytical expressions for the initial stage of MI in the pump+noise case
- Higher-order perturbation solution beyond the first-order linear solution
- Study of the pump power limits of the MI gain
- Tunable Raman (both Stokes and anti-Stokes bands) beyond the pump cuttoff power
- A method to measure the fractional Raman gain $f_R$
- Extension to the GNLSE that allows to model propagation in new materials
Ongoing work

• Extension of the GNLSE, valid for new materials, that includes Raman

• Parametric processes in bulk nonlinear crystals, such as GaSe and AgGaSe$_2$, with a second order nonlinear susceptibility $\chi^2$
  
  ▪ Generation of pairs of correlated photons for quantum information transmission
Thank you!
References: Nonlinear optics


References: Nonlinear optics


References: Robust comp. & info. transmission


References: Memristors

References


• P. I. Fierens, “Number of wireless sensors needed to detect a wild fire”. International Journal of Wildland Fire, Volumen 18, No. 17, pp. 825-829, 2009. DOI: 10.1071/WF07137