Spectral dynamics of noise-seeded modulation instability

International Conference on Applications in Nonlinear Dynamics 2016

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1st September 2016

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CONICET Consejo Nacional de Investigaciones Científicas y Técnicas
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Introduction

Modulation instability

Propagation of a CW in an optical fiber is unstable $\Longrightarrow$ breaks up into pulses

Vast literature

- Benjamin & Feir (1967), Shabat & Zakharov (1972), Akhmediev & Korneev (1986)
- *Optical fibers:* Tai, Hasegawa & Tomita (1986), Potasek (1987)
- **and many more...**

Rekindled interest in MI

Supercontinuum generation

Narrowband input to an optical fiber $\xrightarrow{\textit{nonlinearity}}$ wideband signal

Dudley, Genty & Coen (2006), Dudley & Taylor (eds.) (2010)

Rogue waves

High-amplitude and rare waves that ‘appear from nowhere’

Solli *et al.* (2007), Dudley, Genty & Eggleton (2008)

Anything left unsaid?

40 years of research! Should I end my talk now?

- Most of the analyses of MI do not include all details relevant to optical fibers. One exception: Béjot *et al.* (2011)
- Not a lot of work on (quasi-)analytical approaches to the interaction of noise and nonlinearity in MI

In this talk

- A complete analysis of the spectral evolution of a perturbation to a CW
- First analytical results on input noise + MI
- Some recent results on a noisy input
- Ongoing work...

Applications

What are we interested in?

- Supercontinuum generation in the mid IR
  - Molecular fingerprint region
  - Applications in metrology, tomography, isotope separation, ...
  - Lack of sources in some bands
- Intense pulses - rogue waves
- Parametric amplification

Propagation in optical fibers

Generalized nonlinear Schrödinger equation

$$\frac{\partial A}{\partial z} - i\hat{\beta}A = i\hat{\gamma}A(z,T)\int_{-\infty}^{+\infty} R(T')\left|A(z,T-T')\right|^2 dT', \tag{1}$$

- Dispersion:

$$\hat{\beta} = \sum_{m\geq 2} \frac{i^m}{m!}\beta_m \frac{\partial^m}{\partial T^m}$$

- Nonlinearity:

$$\hat{\gamma} = \sum_{n\geq 0} \frac{i^n}{n!}\gamma_n \frac{\partial^n}{\partial T^n}$$

- Raman scattering: $R(T') = (1-f_R)\delta(T) + f_R h(T)$

Perturbation to the stationary solution

$$A(z,T) = \left(\sqrt{P_0} + a\right) e^{i\gamma_0 P_0 z} = A_s + a e^{i\gamma_0 P_0 z}$$

- Input power: P_0
- Perturbation: $a(z,T)$

Linear terms in the frequency domain

$$\frac{\partial \tilde{a}(z,\Omega)}{\partial z} + \tilde{N}(\Omega)\tilde{a}(z,\Omega) = \tilde{M}(\Omega)\tilde{a}^*(z,-\Omega),$$

- Frequency: $\Omega = \omega - \omega_0$
- $\tilde{N}(\Omega) = -i\left[\tilde{\beta}(\Omega) + P_0\tilde{\gamma}(\Omega)\left(1+\tilde{R}(\Omega)\right) - P_0\gamma_0\right]$
- $\tilde{M}(\Omega) = iP_0\tilde{\gamma}(\Omega)\tilde{R}(\Omega)$

Perturbation to the stationary solution

Ansatz: $a(z,\Omega) = D\exp(iK(\Omega)z)$

$$K(\Omega) = -\tilde{B}(\Omega) \pm K_D(\Omega) = -\tilde{B}(\Omega) \pm \sqrt{\tilde{B}^2(\Omega) - \tilde{C}(\Omega)}$$

- $\tilde{B}(\Omega)$ and $\tilde{C}(\Omega)$ are complex functions of the parameters
- Agrees with Béjot *et al.* (2011)

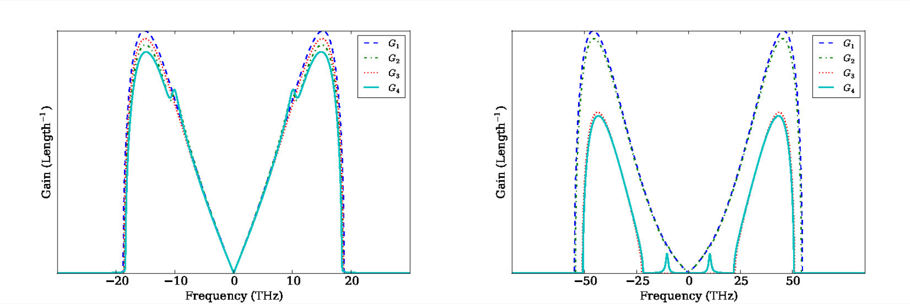
Modulation-instability gain

Only self-steepening: $\gamma_1 = \gamma_0 \tau_{sh}$, $\gamma_n = 0$ for $n \geq 2$

$$K(\Omega) = \tilde{\beta}_o + P_0 \gamma_0 \tau_{sh} \Omega \left(1 + \tilde{R}\right) \pm \sqrt{\left(\tilde{\beta}_e + 2\gamma_0 P_0 \tilde{R}\right) \tilde{\beta}_e + P_0^2 \gamma_0^2 \tau_{sh}^2 \Omega^2 \tilde{R}^2}.$$

- Well-known facts about MI gain $= 2\mathrm{Im}\{K(\Omega)\}$:
  - It does not depend on odd terms of the dispersion relation
  - Self-steepening enables a gain even in a zero-dispersion fiber
  - In the large power limit, it is independent of the dispersion and it is dominated by Raman:

$$|g(\Omega)| \approx 2P_0 \gamma_0 \tau_{sh} |\Omega| \cdot \left| \mathrm{Im}\left\{ \tilde{R}(\Omega) \right\} \right| .$$



Pump power: 100 W (left) and 5 kW (right)

With Raman (G_3 and G_4) and with self-steepening (G_2 and G_4)

Spectral evolution

Spectrum

$$\tilde{a}(z,\Omega) =$$

$$= \frac{e^{-i\tilde{B}(\Omega)z}}{K_D(\Omega)} \cdot \tilde{M}(\Omega)\sin(K_D(\Omega)z)\tilde{a}^*(0,-\Omega)+$$

$$+ \frac{e^{-i\tilde{B}(\Omega)z}}{K_D(\Omega)} \cdot \left[K_D(\Omega)\cos(K_D(\Omega)z) - \left(\tilde{N}(\Omega) - i\tilde{B}(\Omega)\right)\sin(K_D(\Omega)z)\right]\tilde{a}(0,\Omega)$$

- Interaction between $\tilde{a}(0,\Omega)$ and $\tilde{a}(0,-\Omega)$ due to nonlinearity
- $a(0,T) \in \mathbb{R} \implies \tilde{a}(z,\Omega) = \tilde{H}(\Omega,z)\Lambda(\Omega)$

Noise-only input

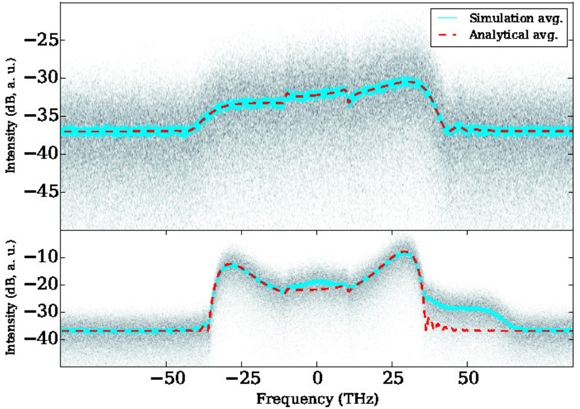
White Gaussian noise

$$\tilde{a}(0,\Omega) \sim \mathcal{CN}(0,\sigma^2) \longrightarrow \tilde{a}(z,\Omega) \sim \mathcal{CN}(0,\sigma_{\tilde{a}}^2)$$

$$\longrightarrow |\tilde{a}(z,\Omega)| \sim \mathrm{Rayleigh}(\sigma_{\tilde{a}}) \longrightarrow |\tilde{a}(z,\Omega)|^2/\sigma_{\tilde{a}}^2 \sim \chi_2^2$$

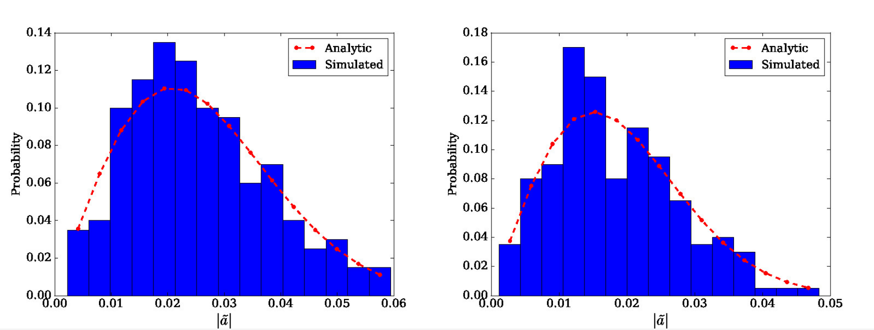
$$\begin{aligned}\sigma_{\tilde{a}}^2 =& \sigma^2 \left|\frac{e^{-i\tilde{B}(\Omega)z}}{K_D(\Omega)}\right|^2 \left\{\left|\tilde{M}(\Omega)\sin(K_D(\Omega)z)\right|^2 + \right.\\ &\left. + \left|K_D(\Omega)\cos(K_D(\Omega)z) - \left(\tilde{N}(\Omega) - i\tilde{B}(\Omega)\right)\sin(K_D(\Omega)z)\right|^2\right\}.\end{aligned}$$

Noise-only input



@ 10 mm (top) and @ 40 mm (bottom)

Noise-only input



Histograms of $|\tilde{a}(z, \Omega)|$ for $z = 10$ mm

$f = 26.758$ THz (left) and $f = -26.758$ THz (right)

Two metrics from supercontinuum generation

Coherence - Dudley, Coen and Genty (2006)

$$g_{12}(z,\Omega) = \frac{\left\langle \tilde{a}_k^*(z,\Omega)\tilde{a}_l(z,\Omega)\right\rangle_{k\neq l}}{\sqrt{\left\langle |\tilde{a}_k(z,\Omega)|^2\right\rangle \left\langle |\tilde{a}_l(z,\Omega)|^2\right\rangle}}$$

- In our setting: $g_{12}(z,\Omega) = 0$

Signal-to-noise ratio - Sørensen *et al.* (2012)

$$\mathrm{SNR}(\Omega) = \frac{\left\langle |\tilde{a}(z,\Omega)|^2\right\rangle}{\sqrt{\mathrm{Var}\left(|\tilde{a}(z,\Omega)|^2\right)}}$$

- In our setting: $\mathrm{SNR}(\Omega) = 1$

Textbook case

Tractable example as a sanity check

$\beta_2 < 0$ (anomalous dispersion), $\beta_k = 0$ for $k > 2$, $\gamma_n = 0$ for $n > 0$, $\tilde{R}(\Omega) = 1$

$$\sigma_{\tilde{a}}^2 = \begin{cases} \sigma^2 \left\{ 1 + \left[\frac{2\left(\frac{\Omega_c}{\Omega}\right)^4}{\left(\frac{\Omega_c}{\Omega}\right)^2 - 1} \right] \sinh^2\left(z \frac{|\beta_2|\Omega^2}{2} \sqrt{\left(\frac{\Omega_c}{\Omega}\right)^2 - 1} \right) \right\} & \Omega < \Omega_c \\ \sigma^2 \left\{ 1 + \left[\frac{2\left(\frac{\Omega_c}{\Omega}\right)^4}{1 - \left(\frac{\Omega_c}{\Omega}\right)^2} \right] \sin^2\left(z \frac{|\beta_2|\Omega^2}{2} \sqrt{1 - \left(\frac{\Omega_c}{\Omega}\right)^2} \right) \right\} & \Omega > \Omega_c \end{cases}$$

$$\Omega_c = 4\gamma_0 P_0 / |\beta_2|$$

Textbook case

Approximation for large z

$$\sigma_{\tilde{a}}^2 \approx \sigma^2 \left\{ 1 + \alpha_z \left[e^{-\frac{(\Omega-\Omega_z)^2}{W_z}} + e^{-\frac{(\Omega+\Omega_z)^2}{W_z}} \right] \right\} \longrightarrow$$

$$r_a(z,\tau) \approx \sigma^2 \left\{ \delta(\tau) + \frac{8\sinh^2\left(\frac{z}{L_{\mathrm{NL}}}\right)}{\sqrt{\pi|\beta_2|z}} e^{-\frac{\tau^2}{4|\beta_2|z}} \cos\left(\frac{\Omega_c}{\sqrt{2}}\tau\right) \right\}.$$

$$L_{\mathrm{NL}} = (\gamma_0 P_0)^{-1}$$

- periodicity $\longrightarrow$ breakup of the CW pump into pulses with a period $\approx \sqrt{2}/\Omega_c$

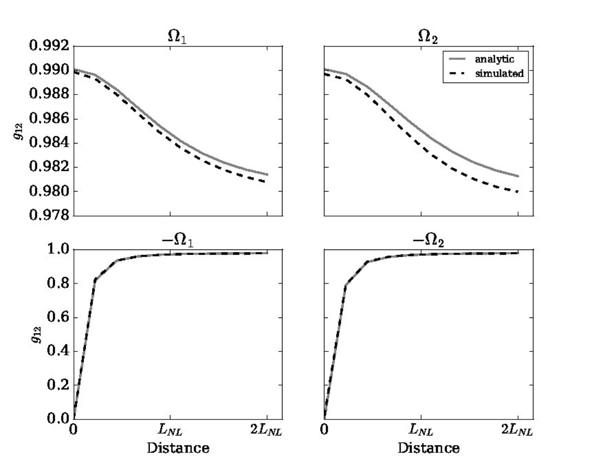
Noisy input

Additive white Gaussian noise

$$a(0,\Omega) = \tilde{s}(\Omega) + \eta(\Omega), \qquad \eta(\Omega) \sim \mathcal{CN}(0, \sigma_a^2)$$

- Relevant for controlling the generation of rogue waves - Solli, Ropers & Jalali (2008); Dudley, Genty & Eggleton (2008); Sørensen *et al.* (2012)
- We developed analytical expressions for the coherence and SNR of the resulting spectrum (accepted at Phys. Rev. A)

Seeded coherence



Seed frequencies: $\Omega_1 = 31$ GHz and $\Omega_2 = 46$ GHz

Seeded coherence

A simple case: One-sided seed, $\tilde{s}(\Omega) = 0$ for $\Omega < 0$

- No self-steepening and no Raman : $\gamma_n = 0$ for $n \geq 1$, $\tilde{R}(\Omega) = 1$
- Net MI gain: $g(\Omega) = 2\mathrm{Im}\{K_D(\Omega)\}$

$z \ll L_{\mathrm{NL}}$

$$g_{12}(z, \Omega) \approx \begin{cases} 1 - \left(1 + \left(\frac{z}{L_{\mathrm{NL}}}\right)^2\right) \frac{\sigma^2}{|\tilde{s}(|\Omega|)|^2} & \Omega > 0, \\ 1 - \left(2 + \left(\frac{L_{\mathrm{NL}}}{z}\right)^2\right) \frac{\sigma^2}{|\tilde{s}(|\Omega|)|^2} & \Omega < 0. \end{cases}$$

$g(\Omega)z \gg 1$

$$g_{12}(z, \Omega) \approx 1 - 2\frac{\sigma^2}{|\tilde{s}(|\Omega|)|^2}$$

Conclusions

So far

- Analytical expressions for the spectral evolution of a perturbation to a continuous pump propagating in an optical fiber, including all relevant effects
- First steps in the analysis of the interaction of noise with the nonlinearity
  - Analytical results for some metrics of supercontinuum generation, such as coherence, for noisy inputs

Ongoing work

- Beyond the undepleted pump approximation
- Influence of seeding on the generation of coherent rogue waves

Questions?

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Perturbation to the stationary solution

Ansatz: $a(z,\Omega) = D\exp(iK(\Omega)z)$

$$K(\Omega) = -\tilde{B}(\Omega) \pm K_D(\Omega) = -\tilde{B}(\Omega) \pm \sqrt{\tilde{B}^2(\Omega) - \tilde{C}(\Omega)}$$

$$\tilde{B}(\Omega) = -\left[\tilde{\beta}_o(\Omega) + P_0\tilde{\gamma}_o(\Omega)\left(1+\tilde{R}(\Omega)\right)\right]$$

$$\tilde{C}(\Omega) = \tilde{\beta}_o^2(\Omega) - \tilde{\beta}_e^2(\Omega) + P_0^2\left(\tilde{\gamma}_o^2(\Omega) - \tilde{\gamma}_e^2(\Omega)\right)\left(1+2\tilde{R}(\Omega)\right) - P_0^2\gamma_0^2 +$$
$$+ 2P_0\gamma_0\tilde{\beta}_e(\Omega) + 2P_0^2\gamma_0\tilde{\gamma}_e(\Omega)\left(1+\tilde{R}(\Omega)\right) + 2P_0\left(\tilde{\beta}_o\tilde{\gamma}_o - \tilde{\beta}_e\tilde{\gamma}_e\right)\left(1+\tilde{R}(\Omega)\right)$$

$$\tilde{\beta}_e(\Omega) = \sum_{n\geq 1}\frac{\beta_{2n}}{(2n)!}\Omega^{2n} \qquad \tilde{\beta}_o(\Omega) = \sum_{n\geq 1}\frac{\beta_{2n+1}}{(2n+1)!}\Omega^{2n+1}$$

$$\tilde{\gamma}_e(\Omega) = \sum_{n\geq 0}\frac{\gamma_{2n}}{(2n)!}\Omega^{2n} \qquad \tilde{\gamma}_o(\Omega) = \sum_{n\geq 0}\frac{\gamma_{2n+1}}{(2n+1)!}\Omega^{2n+1}$$