Recent advances on information transmission and storage assisted by noise

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Introduction

Outline

Information transmission
- In communication systems noise is regarded as a nuisance
- The capacity of a transmission channel is limited by noise

Information storage
- Large scale of integration leads to smaller dimensions and lower voltage levels
- Noise becomes a limiting factor

Memristors
- Resistive RAMs represent one of the most promising candidates for the next generation of computer memories
- Increasing interest in the investigation on the influence of noise
Outline

1. Information transmission assisted by noise
2. Information storage assisted by noise
3. Noisy memristors
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1. Information transmission assisted by noise
2. Information storage assisted by noise
3. Noisy memristors
Noise Ain’t a Nuisance

Information transmission

- Sustained by noise in certain nonlinear channels
  
  García-Ojalvo et al. (2000)

- Noise-assisted fault-tolerant transmission in chains of bistable double-well potentials driven by periodic signals.
  
  Zhang et al. (1998), Perazzo et al. (2000)
Performance characterization

Bit Error Rate

- A relevant performance metric used in digital communication
- A measure of the probability of receiving errors
- For an additive Gaussian noise (AWGN) channel, increasing the SNR decreases the BER

101100101 → Encoder → Channel → Detector → 101000101

PRBS
Information transmission assisted by noise

Bit Error Rate

Single double-well potential

- There is an optimal noise intensity that minimizes the BER

Theory and experiments in VCSELs

Barbay et al. (2000, 2001)
There is a regime where transmission is sustained by noise (sub-critical coupling strengths)

The BER is minimized for an optimal noise intensity even for super-critical coupling strengths
Bit Error Rate

![Graph showing bit error rate vs. noise intensity](image)

- **Input**: Signal with different noise levels.
- **σ < σ_{opt}**: Performance near optimal with less noise.
- **σ_{opt}**: Optimal performance at the noise level.
- **σ > σ_{opt}**: Performance below optimal with increased noise.

**Graph Details**

- **X-axis**: Noise intensity, σ^2
- **Y-axis**: Bit Error Rate (BER)

Legend:
- ○ sub-critical
- □ critical
- △ super-critical
Information transmission assisted by noise

Bit Error Rate

Forward-coupled Schmitt triggers

Patterson et al. (2009)

- STs provide simple models of double-well potentials
- Experimental toy models for the analysis of some forms of regeneration in communication systems

![Diagram showing PRBS, sequential Schmitt triggers, and noise]

PRBS

\[ \text{noise} \]

\[ \text{ST1} \]

\[ \text{noise} \]

\[ \text{ST5} \]

\[ \text{detector} \]
Bit Error Rate

![Bit Error Rate Graph]

- **BER** vs. **noise intensity, $\sigma^2 [V^2]$**
- Key:
  - **ST output**
  - **#1**
  - **#3**
  - **#5**

**Information transmission assisted by noise**
Noise-tunable delay line

Forward-coupled double-well potentials

Ibáñez et al. (2008)

- Delay depends on both noise and coupling strength
- Delay can be noise-tuned even in the super-critical regime
- Application in phase modulation for information transmission
Noise-tunable delay line

The diagrams show the relationship between oscillator number and mean delay for various coupling strengths. The graphs illustrate how the mean delay changes with different noise intensities and coupling strengths, indicating the impact of noise on information transmission assisted by noise.
Outline

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Information storage

- A loop of forward-coupled double-well potentials is able to sustain a traveling wave with the aid of noise

Storing one bit

Two double-well potentials

Ibáñez et al. (2010), Fierens et al. (2010)

- An optimal noise intensity that minimizes the probability of error and maximizes memory persistence
- Experimental results with two STs

 ![Diagram showing writing pulse, double-well potentials, and sub-critical coupling](image)
Storing one bit

\[ \sigma^2 [\Delta U_{01}] \]

\[ \frac{T_m}{T_p} \]

\[ \sigma^2 [\Delta U_{01}] \]

\[ 0.00 \quad 0.05 \quad 0.10 \quad 0.15 \quad 0.20 \quad 0.25 \quad 0.30 \quad 0.35 \quad 0.40 \quad 0.45 \quad 0.50 \]

\[ 0.00 \quad 0.05 \quad 0.10 \quad 0.15 \quad 0.20 \quad 0.25 \quad 0.30 \quad 0.35 \quad 0.40 \quad 0.45 \quad 0.50 \]
Storing multiple bits

Schmitt trigger + Delayed feedback

Bellomo et al. (2011)

- Performance is optimal for an intermediate value of noise intensity
- Probability of error is independent of the number of bits when the elapsed time is normalized to the bit duration
Storing multiple bits

![Graphs showing the probability of error vs. the number of turns and variance](image_url)
1. Information transmission assisted by noise
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Resistive switching

Resistive RAMs

- Some materials change their resistance under the application of electrical pulses
- Resistance may be used to store information: a ‘1’ is represented, say, by a high resistance level and a ‘0’ by a low resistance level
- ReRAMs are one of the most promising candidates for the next generation of computer memories
Noisy memristors

Resistive switching

Memristor

- A two-terminal passive circuit element
  Chua (1971)
- Resistive switching devices are often associated with memristors

\[ v(t) = R(s) \times i(t) \]
\[ \frac{ds}{dt} = \alpha \times F(s) \times i(t) \]

Strukov et al. (2008)
Noisy memristors

Noise Ain’t a Nuisance (?)

Strukov’s model

- The contrast between high and low resistance levels can be enhanced by the addition of internal noise

\[
\frac{ds}{dt} = \alpha \times F(s) \times i(t) + \eta(t)
\]

Stotland and Di Ventra (2012)

- External noise is not useful

\[
\frac{ds}{dt} = \alpha \times F(s) \times (i(t) + \eta(t))
\]

Patterson et al. (2012)
Noisy memristors

Noise Ain’t a Nuisance (!)

(a)

(b)

La$_{0.35}$Pr$_{0.30}$Ca$_{0.375}$MnO$_3$
Conclusions

Information transmission
- Transmission of sub-threshold signals through chains of in-series nonlinear elements is enabled by noise.
- Even supra-threshold signals may benefit from the presence of noise.

Information storage
- Loops of nonlinear elements that work as memory devices only in the presence of noise.

Memristors
- External noise helps to switch resistive states in the presence of a small amplitude driving field.
Information transmission

- Application to high-speed (Gbps) transmission lines
- Influence of other types of noise (e.g., 1/f-noise)

Information storage

- Large scale integration of the proposed memory devices
- Influence of other types of noise (e.g., 1/f-noise)

Memristors

- Models that properly take into account noise
Thanks for listening!

Questions?
References

- G.A. Patterson, P.I. Fierens, D.F. Grosz, in ICAND 2012 - submitted