

Recent advances on information transmission and storage assisted by noise

Pablo I. Fierens

Instituto Tecnológico de Buenos Aires (ITBA)

Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET)

Fourth International Conference on Theory and Applications in
Nonlinear Dynamics

Present & past collaborators

- G. Patterson
- D. Grosz
- A. García
- R. Perazzo
- S. Ibáñez
- G. Bellomo
- A. Goya
- A. Fendrik
- P. Levy

Outline

Information transmission

- In communication systems noise is regarded as a nuisance
- The capacity of a transmission channel is limited by noise

Information storage

- Large scale of integration leads to smaller dimensions and lower voltage levels
- Noise becomes a limiting factor

Memristors

- Resistive RAMs represent one of the most promising candidates for the next generation of computer memories
- Increasing interest in the investigation on the influence of noise

Outline

- 1 Information transmission assisted by noise
- 2 Information storage assisted by noise
- 3 Noisy memristors

Outline

- 1 Information transmission assisted by noise
- 2 Information storage assisted by noise
- 3 Noisy memristors

Noise Ain't a Nuisance

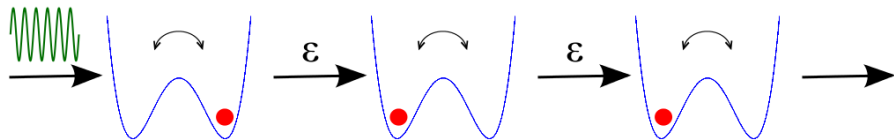
Information transmission

- Sustained by noise in certain nonlinear channels

Lindner *et al.* (1998), Löcher *et al.* (1998), Chapeau-Blondeau (1999), Chapeau-Blondeau and Rojas-Varela (2000), García-Ojalvo *et al.* (2000)

- Noise-assisted fault-tolerant transmission in chains of bistable double-well potentials driven by periodic signals.

Zhang *et al.* (1998), Perazzo *et al.* (2000)



Performance characterization

Bit Error Rate

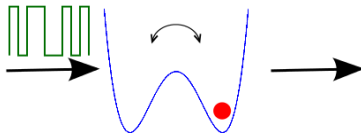
- A relevant performance metric used in digital communication
- A measure of the probability of receiving errors
- For an additive Gaussian noise (AWGN) channel, increasing the SNR decreases the BER



Bit Error Rate

Single double-well potential

- There is an optimal noise intensity that minimizes the BER



- Theory and experiments in VCSELs

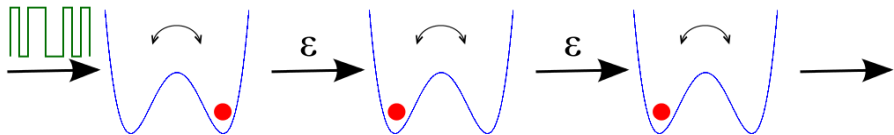
Barbay *et al.* (2000, 2001)

Bit Error Rate

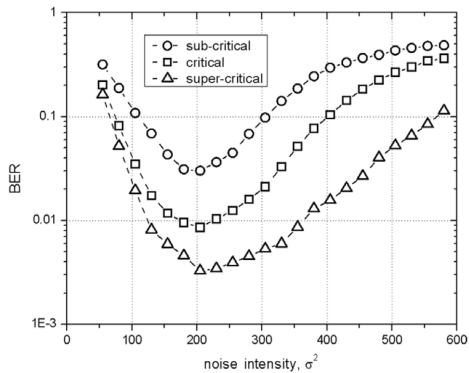
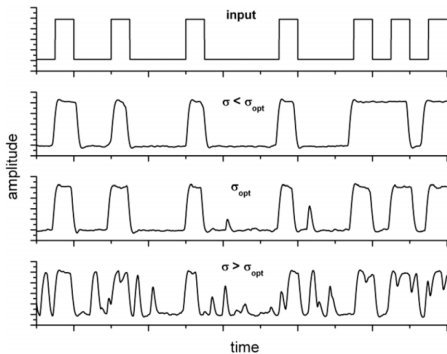
Forward-coupled double-well potentials

Ibáñez *et al.* (2009)

- There is a regime where transmission is sustained by noise (sub-critical coupling strengths)
- The BER is minimized for an optimal noise intensity even for super-critical coupling strengths



Bit Error Rate

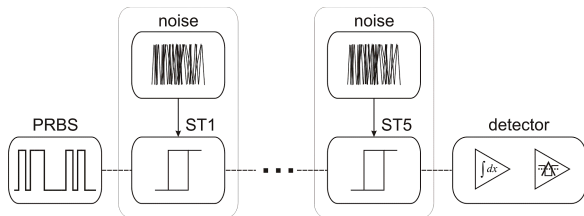


Bit Error Rate

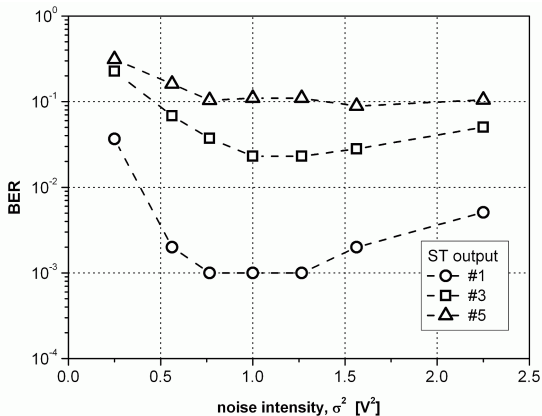
Forward-coupled Schmitt triggers

Patterson *et al.* (2009)

- STs provide simple models of double-well potentials
- Experimental toy models for the analysis of some forms of regeneration in communication systems



Bit Error Rate

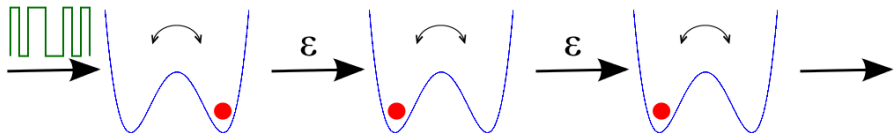


Noise-tunable delay line

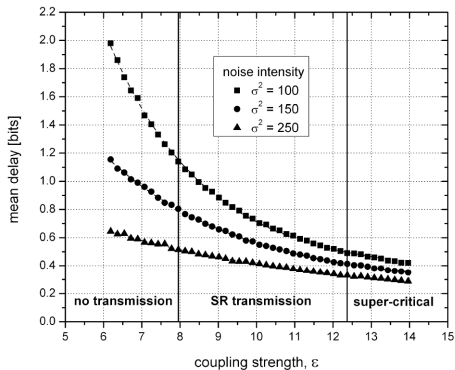
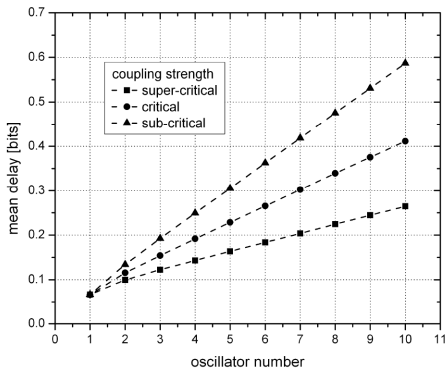
Forward-coupled double-well potentials

Ibáñez *et al.* (2008)

- Delay depends on both noise and coupling strength
- Delay can be noise-tuned even in the super-critical regime
- Application in phase modulation for information transmission



Noise-tunable delay line



Outline

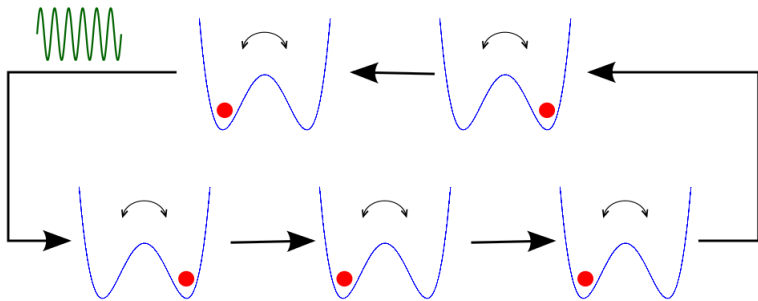
- 1 Information transmission assisted by noise
- 2 Information storage assisted by noise**
- 3 Noisy memristors

Noise Ain't a Nuisance

Information storage

- A loop of forward-coupled double-well potentials is able to sustain a traveling wave with the aid of noise

Carusela *et al.* (2001), Carusela *et al.* (2002)

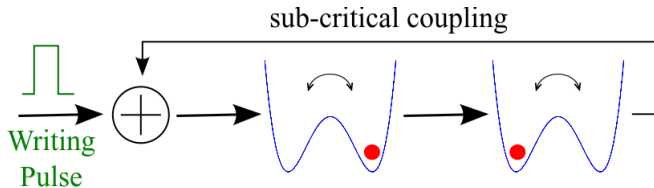


Storing one bit

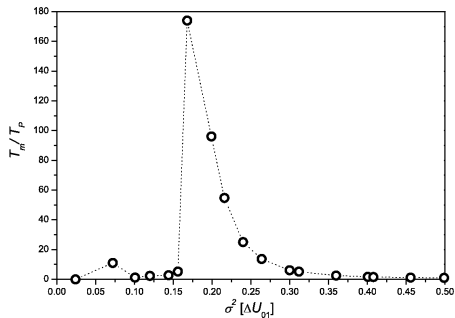
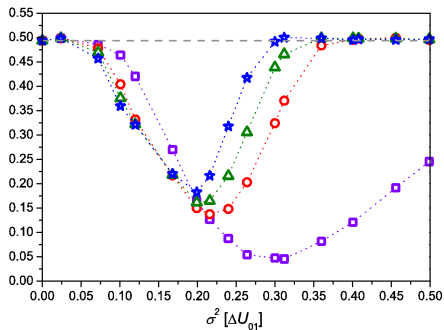
Two double-well potentials

Ibáñez *et al.* (2010), Fierens *et al.* (2010)

- An optimal noise intensity that minimizes the probability of error and maximizes memory persistence
- Experimental results with two STs



Storing one bit

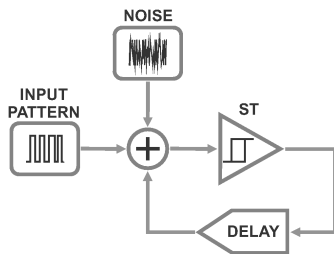


Storing multiple bits

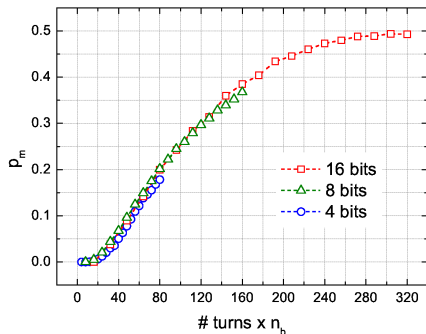
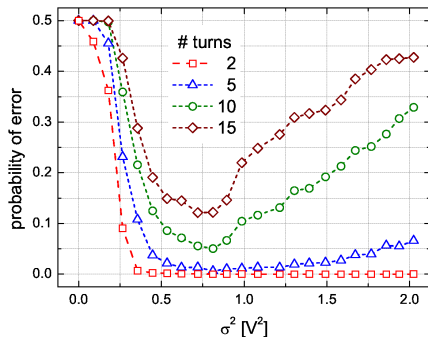
Schmitt trigger + Delayed feedback

Bellomo *et al.* (2011)

- Performance is optimal for an intermediate value of noise intensity
- Probability of error is independent of the number of bits when the elapsed time is normalized to the bit duration



Storing multiple bits



Outline

- 1 Information transmission assisted by noise
- 2 Information storage assisted by noise
- 3 Noisy memristors**

Resistive switching

Resistive RAMs

- Some materials change their resistance under the application of electrical pulses
- Resistance may be used to store information: a '1' is represented, say, by a high resistance level and a '0' by a low resistance level
- ReRAMs are one of the most promising candidates for the next generation of computer memories

Resistive switching

Memristor

- A two-terminal passive circuit element

Chua (1971)

- Resistive switching devices are often associated with memristors

$$v(t) = R(s) \times i(t)$$
$$\frac{ds}{dt} = \alpha \times F(s) \times i(t)$$

Strukov *et al.* (2008)

Noise Ain't a Nuisance (?)

Strukov's model

- The contrast between high and low resistance levels can be enhanced by the addition of internal noise

$$\frac{ds}{dt} = \alpha \times F(s) \times i(t) + \eta(t)$$

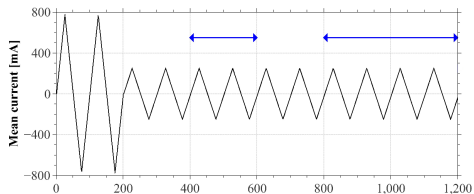
Stotland and Di Ventra (2012)

- External noise is not useful

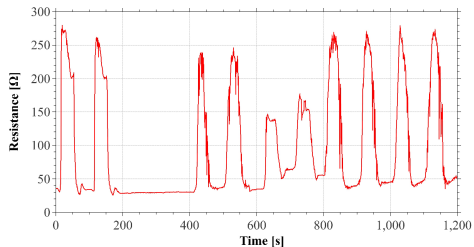
$$\frac{ds}{dt} = \alpha \times F(s) \times (i(t) + \eta(t))$$

Patterson *et al.* (2012)

Noise Ain't a Nuisance (!)



(a)



(b)



Information transmission

- Transmission of sub-threshold signals through chains of in-series nonlinear elements is enabled by noise
- Even supra-threshold signals may benefit from the presence of noise

Information storage

- Loops of nonlinear elements that work as memory devices only in the presence of noise

Memristors

- External noise helps to switch resistive states in the presence of a small amplitude driving field

Information transmission

- Application to high-speed (Gbps) transmission lines
- Influence of other types of noise (e.g., $1/f$ -noise)

Information storage

- Large scale integration of the proposed memory devices
- Influence of other types of noise (e.g., $1/f$ -noise)

Memristors

- Models that properly take into account noise

Thanks for listening!

Questions?

- J.F. Lindner, S. Chandramouli, A.R. Bulsara, M. Löcher, W.L. Ditto, Phys. Rev. Lett. 81(23), 5048 (1998)
- M. Löcher, D. Cigna, E.R. Hunt, Phys. Rev. Lett. 80(23), 5212 (1998)
- Y. Zhang, G. Hu, L. Gammaitoni, Phys. Rev. E 58(3), 2952 (1998)
- F. Chapeau-Blondeau, Electron. Lett. 35, 1055 (1999)
- F. Chapeau-Blondeau, J. Rojas-Varela, Int. J. Bifurcation and Chaos 10, 1951 (2000)
- J. García-Ojalvo, A.M. Lacasta, F. Sagués, J.M. Sancho, Europhys. Lett. 50, 427 (2000)
- R. Perazzo, L. Romanelli, R. Deza, Phys. Rev. E 61(4), R3287 (2000)
- S. Barbay, G. Giacomelli, F. Marin, Phys. Rev. Lett. 85, 4652 (2000)
- S. Barbay, G. Giacomelli, F. Marin, Phys. Rev. E 63(5), 051110 (2001)
- M.F. Carusela, R.P.J. Perazzo, L. Romanelli, Phys. Rev. E 64, 031101 (2001)
- M.F. Carusela, R.P.J. Perazzo, L. Romanelli, Physica D pp. 177-183 (2002)
- S.A. Ibáñez, P.I. Fierens, R.P.J. Perazzo, D.F. Grosz, Physica D 238(21), 2138 (2009)
- G.A. Patterson, A.F. Goya, P.I. Fierens, S.A. Ibáñez, D.F. Grosz, Physica A 389(9), 1965 (2010)
- S.A. Ibáñez, P.I. Fierens, R.P.J. Perazzo, D.F. Grosz, Fluctuation and Noise Lett. 8(3-4), L315 (2008)
- S.A. Ibáñez, P.I. Fierens, R.P.J. Perazzo, G.A. Patterson, D.F. Grosz, Eur. Phys. J. B 76, 49 (2010)
- P.I. Fierens, S. Ibáñez, R.P.J. Perazzo, G.A. Patterson, D.F. Grosz, Physics Letters A 374(22), 2207 (2010)
- G. Bellomo, G. Patterson, P. Fierens, D. Grosz, Physics Letters A 375(37), 3233 (2011)
- L. Chua, IEEE Transactions on Circuit Theory 18(5), 507 (1971)
- D.B. Strukov, G.S. Snider, D.R. Stewart, R.S. Williams, Nature 453, 80 (2008)
- A. Stotland, M. Di Ventra, Phys. Rev. E 85, 011116 (2012)
- G.A. Patterson, P.I. Fierens, D.F. Grosz, in ICAND 2012 - submitted